$$M=\pi\rho b^2[ba(\dot{h}+U\alpha)^{\cdot}-Ub\tfrac{1}{2}\dot{\alpha}-b^2(\tfrac{1}{8}+a^2)\ddot{\alpha}]$$

$$+2\pi\rho Ub^2(a+\tfrac{1}{2})C[(\dot{h}+U\alpha)+b(\tfrac{1}{2}-a)\dot{\alpha}]$$

Writing the loads in this fashion emphasizes that it is not the airfoil section pitch and heave motions (α and h) that must be identified, but rather the mean and linear components of the normal velocity distribution over the airfoil chord. Observe that the $\dot{\alpha}$ terms actually arise from two sources: from the time derivative of $(\dot{h}+U\alpha)$ as well as from the pitch rate. It is worthwhile noting that in this form the result is applicable as well to the case of time-varying freestream velocity U (see Ref. 2), except that the Theodorsen lift deficiency function must be generalized to account for the expansion and compression of the shed wake vorticity. The result is also applicable for arbitrary motion, again except for the lift deficiency function; indeed, the derivation of Ref. 1 does not find it necessary to introduce the assumption of harmonic motion until the final step of evaluating $C(k)$ [Eq. (5-308) of Ref. 1].

The most common approach in helicopter analyses has been to identify $\dot{h}$ as the normal velocity u_P at the rotor blades, and $\dot{\alpha}$ as the pitch rate $\dot{\theta}+\Omega\beta$ (or sometimes even just $\dot{\theta}$). The numerical results for the lift will not be greatly influenced by this error, since the unsteady lift is small compared to the steady component. The steady moment component is normally small however, so the calculation of the moment will be seriously affected if the unsteady terms are incorrect. Another consequence of the incorrect identification of $\dot{h}$ and α, important for the lift as well as for the moment, is that the equivalence of flapping and feathering motion for an articulated rotor blade will be violated.

As an example, consider the rigid flap and rigid pitch motion of an articulated rotor blade in forward flight. Unsteady airfoil theory requires $(\dot{h}+U\alpha)$, which is the air velocity normal to the airfoil section, at the pitch axis; and $\dot{\alpha}$, which is the equivalent pitch rate or camber of the airfoil. Hence for the present case

$$\dot{h}+U\alpha=u_T\theta-u_P=(\Omega r+\Omega R\mu\sin\psi)\theta-(\Omega R\lambda+r\dot{\beta}+\beta\Omega R\mu\cos\psi)$$

$$\dot{\alpha}=\dot{\theta}+\Omega\beta$$

where Ω is the rotor rotational speed, R is the rotor radius, ψ is the blade azimuth angle, r is the radial location, μ is the advance ratio, and λ is the inflow ratio; an arbitrary reference plane is considered. The flap degree of freedom is β, and θ is the pitch degree of freedom. Substitution of these expressions for $(\dot{h}+U\alpha)$ and $\dot{\alpha}$ give the lift and moment in the form required for helicopter analyses. The components due to the 1/rev flap and pitch motion (using $\lambda=\lambda_{tpp}-\mu\beta_{1c}$, where λ_{tpp} is the inflow ratio relative to the tip-path plane) are:

$$\Delta(\dot{h}+U\alpha)=(\Omega r+\Omega R\mu\sin\psi)\,[(\theta_{1c}-\beta_{1s})\cos\psi+(\theta_{1s}+\beta_{1c})\sin\psi]$$

$$\Delta(\dot{\alpha})=-(\theta_{1c}-\beta_{1s})\sin\psi+(\theta_{1s}+\beta_{1c})\cos\psi$$

and similarly for their time derivatives. Therefore, even with the unsteady lift and moment included, the equivalence of flapping and feathering motions is maintained. Specifically, the blade loads depend upon the tilt of the tip-path plane relative to the no-feathering plane ($\beta_{1c}+\theta_{1s}$ and $\beta_{1s}-\theta_{1c}$), independent of the orientation of the rotor shaft.

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References

[1] Bisplinghoff, R.L., Ashley, H., and Halfman, R.L., *Aeroelasticity*, Addison-Wesley Publishing Co., Reading, Mass., 1955.

[2] Greenberg, J.M., "Airfoil in Sinusoidal Motion in a Pulsating Stream," NACA TN 1326, June 1947.

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Class of Shockfree Airfoils Producing the Same Surface Pressure

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Introduction

THE design of shockfree airfoils is critical to transonic aerodynamics because shockwaves, which are associated with entropy rises that produce wave drag, also contribute significantly to boundary-layer separation and performance degradation. The existence of shockfree flows is therefore a subject of intense engineering as well as mathematical interest. However, the existence question is many-faceted: different "host" formulations are possible and each provides different information vital to a complete understanding. Taking one approach Morawetz,[1] in the middle 1950's, proved that planar shockfree flows are mathematically isolated in that arbitrary changes to the flow or boundary conditions providing a smooth flow lead to shock formation. This result was recently extended to three dimensions by Cook.[2] On the other hand, Sobieczky[3] and Sobieczky, et al.,[4] recently showed that in two dimensions there exist, for any small change in freestream Mach number, an infinite number of small changes in airfoil shape that will ensure a shockfree flow. In their approach, the required shape changes are selected by a "fictitious gas" scheme embedded in a mixed-type relaxation algorithm[4]; note, however, that the host boundary-value problems used in generating their shockfree airfoils offer no control over the form of the resulting surface pressure distribution. The present paper addresses still another aspect of the general problem, namely, the properties of those airfoils obtained over a wide range of cruise Mach numbers that induce the same fixed supercritical shockfree surface pressure.

The question raised above is interesting for the following reasons. The flow past a fixed airfoil, assuming constant density, is described by a boundary-value problem for the disturbance velocity potential $\phi(x,y)$, namely, $\phi_{xx}+\phi_{yy}=0$, with ϕ_y specified on a chordwise slit, $\nabla\phi\to 0$ at infinity, and a jump in potential $[\phi]$ chosen to satisfy Kutta's requirement for smooth flow from the trailing edge. (x and y are streamwise and transverse coordinates.) On the other hand, the airfoil which produces a prescribed chordwise surface pressure can be found from the solution to a boundary-value problem for the disturbance stream function $\psi(x,y)$. The equation $\psi_{xx}+\psi_{yy}=0$ is solved with ψ_y specified on an approximating slit, noting that $\psi_y=\phi_x=-\tfrac{1}{2}C_p$, where C_p is the surface pressure coefficient, $\psi\to 0$ at infinity, and a jump in stream function $[\psi]=0$ along the downstream slit emanating from the trailing edge to enforce closure (the surface elevations are obtained from $\mathrm{d}y/\mathrm{d}x=-\psi_x(x,0)$). Because the two formulations are mathematically equivalent, pathological behavior obtained in analysis solutions for ϕ would find their counterpart in design solutions for ψ. Now, for transonic supercritical flow, Morawetz states that small changes in the subsonic freestream Mach number corresponding to a smooth flow lead to discontinuities in surface ϕ_x (unless, of course, the airfoil is simultaneously altered using Sobieczky's procedure); thus, one would suspect that off-design airfoils generated from an inverse procedure

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by holding a prescribed shockfree surface pressure fixed and varying the Mach number might involve discontinuities in ψ_x or surface slope.

Thus, in this paper we consider the inverse counterpart of Morawetz's question: Do realistic airfoils exist given a shockfree surface pressure fixed throughout a range of operating Mach numbers? For linearized flows, obviously, the equivalence between ϕ and ψ formulations leads to a number of interesting dualities relating the design problem for thickness to the analysis problem for camber and the analysis problem for thickness to the design problem for camber (e.g., see Chin[5].) However, for flows near Mach 1, these dualities break down because of the transonic nonlinearity. General conclusions are not available. Therefore, in the following work we present numerical results for a series of airfoils corresponding to a known baseline shockfree pressure and show that physically acceptable airfoils can be found. Discontinuities in surface slope, which may have been anticipated (as previously discussed) in fact did not appear for the large Mach number range considered, while surface crossovers, which are not disallowed in the mathematical fromulation, appeared only at extreme off-design conditions. These results are significant, of course, in the engineering of "adaptive wings" aimed at fixing, throughout a range of Mach numbers M, pressure distributions with desirable loading and boundary-layer separation characteristics.

Analysis

The steady transonic supercritical flow past a thin airfoil can be obtained from a "direct" formulation solving $(1-M^2-M^2(\gamma+1)\phi_x)\phi_{xx}+\phi_{yy}=0$, γ being the ratio of specific heats, specifying $\phi_y(x,0\pm)$ on an approximating chordwise slit, jump conditions at the shock, regularity conditions at infinity, and a jump in potential $[\phi]$ chosen, as before, to fulfill Kutta's requirement for smooth flow from the trailing edge. (Surface pressure coefficients are obtained from $C_p=-2\phi_x$.) This boundary-value problem and the fundamental type-dependent relaxation method which underlies its solution are discussed at length by Cole.[6] The "inverse" calculation can also be solved in a "direct" manner, as in the constant density case, by formulating the problem through a disturbance stream function ψ. A classic sign ambiguity connected with the appearance of two velocities ϕ_x, one subsonic and the other supersonic, for each prescribed streamwise mass flux ψ_y, has in the past discouraged workers from stream function approaches. Recently, however, Chin and Rizzetta[7] bypassed this difficulty by reformulating the problem through a mixed-type integro-differential equation for ψ. The resulting inverse formulation for ψ complementing the foregoing analysis formulation for ϕ is exactly:

$$\left[1-M^2+M^2(\gamma+1)\int_{\pm\infty}^{y}\psi_{xx}(x,s)\,ds\right]\psi_{xx}+\psi_{yy}=0$$

$\psi_y(x,\pm 0)=-\frac{1}{2}(1-M^2)C_p-\frac{1}{8}M^2(\gamma+1)C_p^2$ on the chord

$\psi\to 0$ at infinity

$[\psi]=0$ along the downstream wake and, jump conditions conserving mass and vorticity at the shock

(Note that ϕ and ψ are related through $\psi_y=(1-M^2)\phi_x-\frac{1}{2}M^2(\gamma+1)\phi_x^2$ and $\psi_x=-\phi_y$.) This may be regarded as a Neumann boundary-value problem for ψ, not dissimilar to Cole's Neumann problem for ϕ. The ψ formulation can be similarly solved, thus, using a type-dependent column relaxation procedure employing tridiagonal matrices, where all matrix coefficients, including the integral-like coefficient, are evaluated with latest available values. In Ref. 7 the authors demonstrate the utility of the approach by recovering the correct airfoils, in the first case, given a shockfree Korn pressure input, and, in the second case, given a Jameson surface pressure distribution with a strong shock. The numerical details and the relevant theory, discussed at length in Ref. 7, will not be redescribed here.

The supercritical surface pressure distribution corresponding to a shockfree Korn airfoil section 75-06-12 at Mach 0.75 and zero angle of attack was used as the baseline input pressure; the upper and lower surface C_p shown in Fig. 1 were obtained using the Garabedian-Korn transonic analysis code, which solves the full potential equation on a body-fitted curvilinear mesh. These surface C_p's were used on $y=0$ (involving some error, of course, but exact shape recovery is not important for our purposes) and a coarse 3×4 chord Cartesian mesh with 60×60 nonvarying grids, 20 taken between the leading and trailing edges, was assumed. Airfoil ordinates were obtained from $dy/dx=\phi_y(x,0)$ or $y^{\pm}(x)=-\psi(x,0\pm)$ to within a constant. The completely converged solution obtained after "sweeping" the computational flowfield 1000 times is shown in Fig. 2 for $M=0.75$. Also shown are airfoils obtained for a wide range of off-design Mach numbers in the range of 0.72-0.79. All of these shapes are physically acceptable except, however, that generated using 0.79, which shows aft surface crossover. With increasing M, the section shapes show an increasing droop associated with a reduction in the angle of attack, the expected decrease in thickness ratio, complete smoothness in surface definition, and, surprisingly, little change in the lower surface aft curvature. These results may suggest that smooth realistic airfoils may likely be found at any Mach number. Of course, this conclusion holds only for the particular pressure distribution assumed here and, then, only for those Mach numbers considered.

Discussion and Conclusion

The family of airfoils shown in Fig. 2 is made possible by the recent scheme of Chin and Rizzetta[7] which provides a "direct" solution of the transonic inverse problem. Our results indicate that, for the inverse problem, neighboring airfoil solutions can be completely smooth and completely realistic. From Fig. 2, it appears that inverse solutions may be stable to large Mach number perturbations in the sense that "shockwaves," here related to discontinuities in surface slope (which were anticipated from the dual analogy to analysis

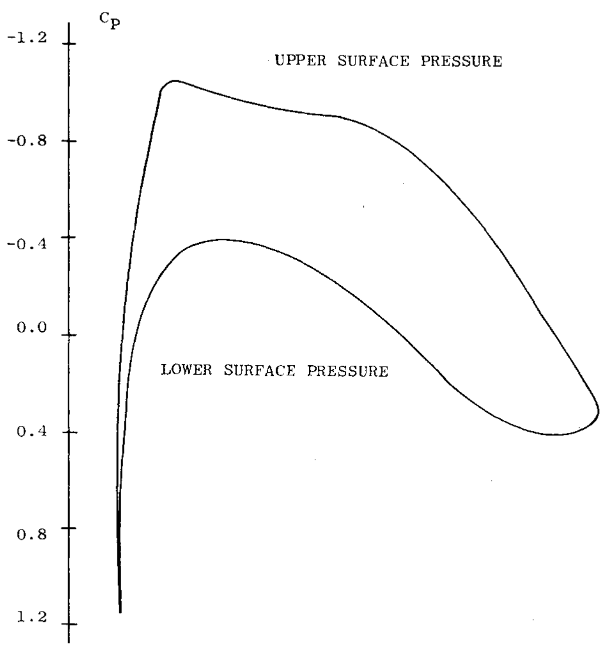

Fig. 1 Input shockfree surface pressure distribution.

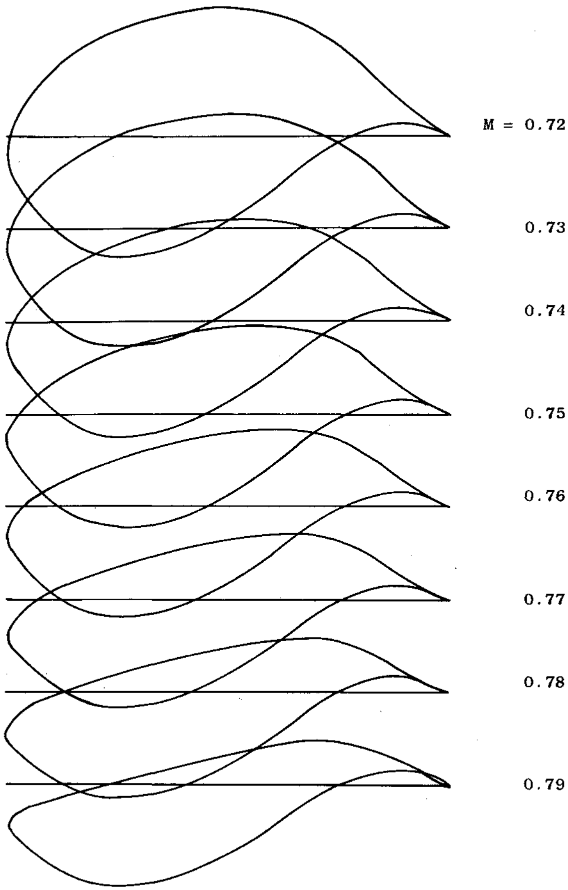

Fig. 2 Airfoils obtained by varying Mach number (vertical scale magnified five times).

problems), need not appear. However, further mathematical investigation is needed.

The idea of an "adaptive airfoil" capable of adjusting itself so as to maintain a predetermined shockfree surface pressure is intriguing. For small changes about a design Mach number, Fig. 2 shows that the lower surface geometry including the aftward curvature remains more or less unchanged, with the basic alterations appearing in the upper surface; the adaptive surface, in most cases, will be the upper one. The present results also suggest a new definition of "optimum airfoils." An optimum shape is one that operates best over a range of cruise conditions and, in engineering practice, is selected (rather arbitrarily) out of many possible shapes. It is possible that the designer may wish to fix the pressure distribution over a small range of M, e.g., choose that shown in Fig. 1. If so, the optimum shape can be defined as an average of the coordinates displayed in Fig. 2 weighted, of course, against the time T_M actually spent in flying at a particular Mach number M. The usefulness of the "direct" inverse formulation discussed in Ref. 7 is stressed in this paper: first, as a practical design tool allowing direct control of loading and boundary-layer separation characteristics; and, second, as a mathematical vehicle through which the nature of transonic inverse solutions may be more completely understood.

References

[1] Morawetz, C.S., "On the Non-Existence of Continuous Transonic Flows Past Profiles, I, II and III," *Communications on Pure and Applied Mathematics,* Vols. 9, 10, and 11, 1956, 1957, and 1958, pp. 45-68, 107-131, and 129-144.

[2] Cook, L.P., "A Uniqueness Proof for a Transonic Flow Problem," *Indiana University Mathematical Journal,* Vol. 27, Jan.-Feb. 1977, pp. 51-72.

[3] Sobieczky, H., "Die Berechnung lokaler Räumlicher Überschallfelder," Lecture at Gesellschaft für angewandte Mathematik und Mechanik, Copenhagen, May-June 1977, ZAMM 58T, 1978, pp. 215-216.

[4] Sobieczky, H., Yu, N.J., Fung, K.Y., and Seebass, A.R., "New Method for Designing Shock-free Transonic Configurations," *AIAA Journal,* Vol. 17, July 1979, pp. 722-729.

[5] Chin, W.C., "On the Design of Thin Subsonic Airfoils," *ASME Journal of Applied Mechanics,* Vol. 46, March 1979, pp. 6-8.

[6] Cole, J.D., "Modern Developments in Transonic Flow", *SIAM Journal of Applied Mathematics,* Vol. 29, Dec. 1975, pp. 763-787.

[7] Chin, W.C. and Rizzetta, D.P., "Airfoil Design in Subcritical and Supercritical Flows," *ASME Journal of Applied Mechanics,* Dec. 1979, pp. 761-766.